11.3 Compound Interest

**Investment** – use of money or capital for profit.

**Fixed investment** – amount invested as principal is guaranteed and interest is computed at a fixed rate. (ex. Savings accounts, certificates of deposits, government savings bonds)

**Variable investment** – neither the principal nor the interest is guaranteed. (ex. Stocks, mutual funds, commercial bonds)
Compound interest – interest that is computed on the principal and any accumulated interest.

\[ A = p(1 + \frac{r}{n})^{nt} \]

with \( A \) = amount at time \( t \)

\( p \) = principal

\( r \) = **annual** rate of interest

\( n \) = number of periods/year

\( t \) = number of years
Effective annual yield (or annual percentage yield APY) is the simple interest rate that gives the same amount of interest as a compound rate over the same period of time.

[Compute \(1(1+\frac{r}{n})^{n(1)} - 1\): note that \(p = 1\) and \(t = 1\).]
If you want to have a certain amount of money $A$ in $t$ years, the amount $p$ which would have to be invested now is called the **present value**.

$$\displaystyle p = \frac{A}{(1+\frac{r}{n})^{nt}}$$

**with**

- $p = \text{present value}$
- $A = \text{amount of money required in the future}$
Examples:

Use \( A = P \left(1 + \frac{r}{n}\right)^{nt} \) to compute:

1. The amount of the investment if $3000 is invested for 5 years at 5% compounded annually.

\[
\begin{align*}
A &= P \left(1 + \frac{r}{n}\right)^{nt} \\
&= 3000 \left(1 + \frac{0.05}{1}\right)^{5} \\
&= 3000 \left(1 + .05\right)^{5} \\
&= 3000 \left(1.05\right)^{5} \\
&= 3000 \times 1.27628 \\
&= \$3828.84
\end{align*}
\]

Interest earned:

\[
3828.84 - 3000 = \$828.84
\]
2. The amount of the investment if $5000 is invested for 10 years at 6.75% compounded daily (n=360).

\[ p = 5000 \]
\[ n = 360 \]
\[ r = 6.75\% = .0675 \]
\[ t = 10 \]
\[ A = p \left(1 + \frac{r}{n}\right)^{nt} = 5000 \left(1 + \frac{.0675}{360}\right)^{360(10)} \]
\[ = \boxed{9819.54} \]
3. The effective annual yield if money is invested at 7.5% compounded monthly.

\[ \text{APY} = \left(1 + \frac{r}{n}\right)^n - 1 \]

\[ p = 1 \]
\[ t = 1 \]
\[ r = 7.5\% = 0.075 \]
\[ n = 12 \]

\[ \text{APY} = \left(1 + \frac{0.075}{12}\right)^{12} - 1 \]

\[ = 1.077633 - 1 \]

\[ = 0.077633 \]

\( \text{or} \ 7.8\% \)
4. Buddy wants to invest some money now to buy a new tractor in the future. If he wants to have $30,000 available in 5 years, how much does he need to invest now in a CD paying 5.15% interest compounded monthly?

\[ P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} \]

\[ A = 30,000 \]
\[ t = 5 \]
\[ r = 5.15\% = 0.0515 \]
\[ n = 12 \]

\[ P = \frac{30,000}{\left(1 + \frac{0.0515}{12}\right)^{60}} \]

\[ = \frac{30,000}{\left(1 + 0.0042916666\right)^{60}} \]

\[ = \frac{30,000}{1.292979} \]

\[ = 23,202.23 \]
11.4 Installment Buying

**Open-end installment loan** is a loan on which you can make variable payments each month. (ex. Credit cards)

**Fixed installment loan** is one on which you pay a fixed amount of money for a set number of payments. (ex. Student loans, car loans) *(similar to APY)*

**Annual percentage rate (APR)** is the true rate of interest charged for the loan.
Finance charge is the total amount of money the borrower must pay for its use. (Includes interest plus any additional fees.)

Total installment price is the sum of all the monthly payments and the down payment, if any.
Fixed Installment Loans

Example:

1. Remodeling a Living Room. Oda Lisa Hernandez received a bid of $4200 to remodel her living room. To finance this amount, her savings and loan requires her to pay 15% down, with the balance being financed with a 24-month installment loan with an APR of 7.5%.
   a.) Determine Oda’s total finance charge.
   b.) Determine Oda’s monthly payment.

\[
\text{down payment} = 15\% \text{ of } 4200
\]
\[= 0.15(4200) = 630.\]
\[
\text{amount financed} = 4200 - 630
\]
\[= 3570.\]
To compute finance charge, use Table 11.2 on p. 623

<table>
<thead>
<tr>
<th>Number of Payments</th>
<th>APR 7.5</th>
<th>APR 9.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>8.00</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Oda's finance charge = \( \frac{\text{amt financed}}{100} \times 8.00 \)

\[ = \frac{3570}{100} \times 8.00 = 285.6 \]

Total to be repaid = \( \text{amt financed} + \text{finance charge} \)

\[ = 3570 + 285.6 = 3855.6 \]

Monthly payment = \( \frac{3855.60}{24} \)

\[ = 160.65 \]
3. Average Daily Balance Method. The Levy’s card statement shows a balance due of $1578.25 on March 23, the billing date. For the period ending April 23, they had the following transactions:

<table>
<thead>
<tr>
<th>Date</th>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 26</td>
<td>Charge: Party supplies</td>
<td>$79.98</td>
</tr>
<tr>
<td>March 30</td>
<td>Charge: Restaurant meal</td>
<td>52.76</td>
</tr>
<tr>
<td>April 3</td>
<td>Payment</td>
<td>250.00</td>
</tr>
<tr>
<td>April 15</td>
<td>Charge: Clothing</td>
<td>190.52</td>
</tr>
<tr>
<td>April 22</td>
<td>Charge: Car repairs</td>
<td>190.85</td>
</tr>
</tbody>
</table>

a.) Find the average daily balance for the billing period.

b.) Find the finance charge to be paid on April 23. Assume an interest rate of 1.3% per month.

c.) Find the balance due on April 23.
<table>
<thead>
<tr>
<th>DATE</th>
<th>CHARGE</th>
<th>PAYMENT</th>
<th>BALANCE</th>
<th>NO. OF DAYS</th>
<th>BALANCE X NO. OF DAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mar. 23</td>
<td></td>
<td></td>
<td>1578.25</td>
<td>3</td>
<td>4734.75</td>
</tr>
<tr>
<td>Mar. 26</td>
<td>79.98</td>
<td></td>
<td>1658.23</td>
<td>4</td>
<td>6632.92</td>
</tr>
<tr>
<td>Mar. 30</td>
<td>52.76</td>
<td></td>
<td>1710.99</td>
<td>4</td>
<td>6843.96</td>
</tr>
<tr>
<td>Apr. 3</td>
<td></td>
<td>250.00</td>
<td>1460.99</td>
<td>12</td>
<td>17531.88</td>
</tr>
<tr>
<td>Apr. 15</td>
<td>190.52</td>
<td></td>
<td>1651.51</td>
<td>97</td>
<td>11560.57</td>
</tr>
<tr>
<td>Apr. 22</td>
<td>196.85</td>
<td></td>
<td>1842.36</td>
<td>1</td>
<td>1842.36</td>
</tr>
<tr>
<td>Apr. 23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>49146.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Add up entries in col. 6

Average daily balance = \( \frac{49146.44}{31} \)

= \$1585.37

Finance charge

\[ i = \text{prt} \]

\[ i = (1585.37)(0.013)(1) \]

= \$20.61

Balance due = \( \frac{\text{last balance} + \text{interest}}{\text{last balance} + \text{interest}} \)

= 1842.36 + 20.61

= \$1862.97